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Journal Item

Teaching Mathematics and its Applications: An International Journal of the IMA (2021) **40**, 210–233
https://doi.org/10.1093/teamat/hrab004 Advance Access publication 22 March 2021

Further Mathematics, student choice and transition to university: part 2—non-mathematics STEM degrees

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[Received November 2019; accepted February 2021]

This is the second paper reporting the results of a study investigating student choices of optional post-16 advanced-level (A-level) Mathematics and Further Mathematics qualifications in the UK and their impact on the transition from school to university mathematics. Here, the opinions of non-mathematics Science, Technology, Engineering and Mathematics (STEM) undergraduate students (all of whom had previously studied A-level Mathematics) were accessed via a survey and individual interviews. The study found that Further Mathematics qualifications are perceived as advantageous for non-mathematics STEM degrees by students once they are at university but not when making A-level choices. While the students often perceived mathematics positively, this appears to influence the choice of A-level Mathematics but not Further Mathematics. The lack of support from teachers and parents, the lack of perceived utility of Further Mathematics qualifications and a perception that Further Mathematics is only useful for studying a mathematics degree could all be factors affecting the uptake of Further Mathematics. The identified perceived impact of Further Mathematics on the university transition is linked to studying more pure mathematics which may give students a better understanding of how to apply mathematics in the context of their degree.

Some comparisons between the findings in Parts 1 and 2 of the study are included which suggests that the Further Mathematics qualification is better serving students intending to study a non-mathematics STEM degree rather than mathematics itself.

1. Introduction

It is recognized that the mathematics students learn at school will form one of the ‘two pillars’ (together with subject specific knowledge) on which future success in scientific disciplines is built (Sadler & Tai, 2007). However, despite its fundamental importance, studying mathematics is only compulsory up to the age of 16 years in some countries such as the UK. Consequently, the choice of how much mathematics to study post-16 has emerged as an area of research with particular focus on the transition from school to university in Science, Technology, Engineering and Mathematics (STEM) subjects. One may argue that it is the responsibility of universities to clearly signpost students to study more mathematics via their entry requirements, but it has been reported that this is often not the case (see, e.g., ACME, 2011; Hillman, 2014; Hodgen *et al.*, 2014), with some university courses not requiring students to study mathematics post-16 for admission, despite involving a substantial mathematical content. Moreover, even when students do take mathematics qualifications post-16, studies question how well these qualifications prepare them for studying at university (Hawkes & Savage, 1999; ACME, 2011; Engelbrecht *et al.*, 2012; Flegg *et al.*, 2012; Matic, 2014) with criticisms of both their content and how the mathematics is taught and subsequently learnt by the students.

The present research is concerned with the choices students make about the mathematics they study during their final years of secondary education and how they perceive these choices impact upon their transition to STEM university degrees. The study is set in the UK, within the context of England and Wales. The UK is made up of four separate nations (England, Northern Ireland, Scotland and Wales) with the UK government responsible for education policy in England but with devolved administrations responsible in the other three nations (Forsythe & Smith, 2020). We focus on England and Wales together as they share a common structure to their qualifications with students typically studying for General Certificate of Secondary Education (GCSE) qualifications in a range of subjects between the ages of 14 and 16 years (which must include Mathematics) and then studying for advanced-level (A-level) qualifications in three or four subjects between the ages of 16 and 18 years (which do not need to include Mathematics). While the A-level has retained its name since the 1950s, significant alterations have been made to the content and structure of the qualifications. Initially, mathematics was offered as two separate A-levels of equal difficulty (pure mathematics and applied mathematics) but was changed through the 1960s and 1970s to a hierarchical pair (Mathematics and Further Mathematics) each involving both pure and applied topics with Further Mathematics only available to students who are also studying Mathematics. It has been argued that these two maths A-levels do not well serve students considering degrees such as social sciences, geography or business (Browne *et al.*, 2013). A new qualification called Core Maths was introduced in 2014 to fill this gap and was envisaged to serve such students by focussing on applied aspects and problem solving. This was not a new idea as an alternative applied mathematics qualification Use of Mathematics had been previously introduced almost two decades earlier (Hutcheson *et al.*, 2011). However, this did not become a mainstream alternative to Mathematics A-level and a recent study by McAlinden & Noyes (2019) implied that considerable effort from both government and universities would be needed for ‘Core Maths’ to avoid the same destiny.

While A-level Mathematics is the most popular of all A-level qualifications, only a small proportion of students (about 17%) who take A-level Mathematics also take Further Mathematics A-level (JCQ, 2017) and Further Mathematics remains one of the least popular A-level qualifications. Over the past 20 years, there have been noted problems with the provision of Further Mathematics across England and Wales due to a range of problems including (but not limited to) a shortage of skilled mathematics teachers, low takeup of the qualification and financial constraints on schools (see Tanner *et al.*, 2016, for a detailed discussion). To address these issues, the UK government established the Further Mathematics Support Programme (FMSP) in 2003 to strengthen the provision of A-level Further Mathematics across England through supporting schools to provide the qualification and offering blended online learning where this is not possible. The concept was also adopted by the Welsh Government in 2010 with the establishment of the FMSP Wales. Since the establishment of these programmes, there has been a substantial increase in uptake of Further Mathematics across both England and Wales (Tanner *et al.*, 2016; Lyakhova & Neate, 2019).

Within this context, we focus on how students choose to study A-level Mathematics and how they decide whether or not to study Further Mathematics before going on to study a non-mathematics STEM degree. We investigate the A-level qualifications before the latest set of reforms (ALCAB, 2014) were implemented in 2017–2018. At the time of the study, both maths A-levels were modular qualifications with a great deal of flexibility in content, particularly in terms of applied content (mechanics, statistics or both) which allowed students to specialize in one area of application. Students were also able to study only the first half of an A-level course which would result in an advanced subsidiary-level (AS-level) qualification. The intention of the latest set of reforms was to incorporate a greater amount of problem solving and modelling into both mathematics A-levels. But the reformed qualifications are far less flexible in content and students only have the opportunity to study more than one module in either mechanics or statistics if they take at least AS-level Further Mathematics in addition to A-level Mathematics. It has been proposed that this may lead to more STEM students taking Further Mathematics (see, e.g., an argument in Bowyer & Darlington, 2016).

In this study, we used both a survey and individual interviews of undergraduate students studying for STEM degrees to investigate how they chose to study A-level Mathematics and how they decided whether or not to additionally study A-level Further Mathematics. This is the second of two papers and presents the results from those undergraduate students who were at university studying for a non-mathematics STEM degree and a comparison with the findings on undergraduate students who were studying for a mathematics degree which were reported in the first paper (Lyakhova & Neate, 2019).

2. The transition to university degree courses with some mathematical content

The UK university sector is very diverse with distinct groupings of institutions based on their age and performance in a range of areas including research activity, teaching quality and academic selectivity for admissions (Boliver, 2015). In particular, there are distinctions between traditional research intensive universities and the more modern teaching-focussed universities where many of the former are more academically selective for admissions. Among the research intensive universities, the 'Russell Group' is a grouping of 24 universities who have been very successful at promoting themselves as the UK's leading institutions.

It has been noted that in the UK, a large number of undergraduate students did not study an adequate amount of mathematics in the last 2 years of school (ACME, 2011). For example, in 2010, only 64% of first year undergraduate chemistry students studied A-level Mathematics (Hillman, 2014). A study by Darlington & Bowyer (2016a) of engineering undergraduate students found that those who took A-level Further Mathematics in addition to A-level Mathematics found Further Mathematics a good preparation. Yet, the data (as reported in 2011) showed that only 19% of first year engineering students took the qualification (Vidal Rodeiro *et al.*, 2013). Similarly, only 48% of undergraduate students who entered computer-related university degrees in 2013 had studied AS- or A-level Mathematics (Hodgen *et al.*, 2014). Furthermore, a report published in 2011 identified that as much as 50% of the undergraduate student cohort had not studied mathematics after the age of 16 years and would have benefited from doing so (ACME, 2011). This is worrying as a lack of prior mathematical skills and knowledge is noted to have negative effects on the performance of undergraduate students, while studying more mathematics before entering university is associated with better performance at degree level (see, e.g., Gadd, 2000; Porkess, 2007; Lee *et al.*, 2008; Lee & Lee, 2009; Asshaari *et al.*, 2012; Hermon & Cole, 2012; Güner, 2013).

There is a large spectrum of difficulties identified for students in both learning and applying mathematics at university. Undergraduate students are known to struggle not only with basic arithmetic and algebraic manipulation but also with applying known mathematical ideas and methods in unknown situations to new problem areas or in unfamiliar contexts (IOP, 2011; Koenig, 2011; Hodgen *et al.*, 2014). This is sometimes argued to stem from the fact that school mathematics is seen as repetitive and predictable (Darlington & Bowyer, 2016a) with students trained to answer particular types of questions. More generally, some studies report that students have unrealistic expectations about how much mathematics is involved in studying STEM subjects at university (King & Cattlin, 2014), which could create a feeling of anxiety in new undergraduate students (Gadd, 2000; IOP, 2011; Shallcross & Yates, 2014).

Several studies mention A-level Mathematics as helpful for studying mathematics as part of a non-mathematics degree course (Edwards, 1995; Lee *et al.*, 2008; Hermon & Cole, 2012). On the other hand, regardless of whether their university required A-level Mathematics or not, STEM students are known to encounter more mathematics content at university than they expected (see, e.g., Forsey *et al.*, 2001; IOP, 2011) indicating that A-level Mathematics may not be an ideal preparation. In such contexts, it has been reported that studying Physics post-16 may also be associated with better performance for STEM degree courses (Hermon & Cole, 2012) due to providing extra experience of studying mechanics topics and mathematical modelling (see, e.g., an argument by Darlington and Bowyer, 2016b). Further Mathematics qualifications were also found to be perceived as helpful for studying a STEM degree.

When surveying undergraduate chemistry, engineering and physics students, Darlington and Bowyer (2016a,b) found that students who took Further Mathematics felt positive about it and described it as a good preparation for their courses in all disciplines. Pure mathematics topics covered in Further Mathematics were reported as very useful or most useful in all three studies and many topics that students suggested for inclusion in the pre-university courses for either chemistry or engineering were, in fact, Further Mathematics topics. This may reflect the fact that students met Further Mathematics topics in the first year of their degree. In addition, the physics undergraduates perceived their familiarity with the content especially useful because at university, the material was covered more quickly and required more independent work than at school. The study of chemistry students noted that some students proposed that Further Mathematics should be made compulsory for the admission to their degree courses and proposed that university departments might consider recommending Further Mathematics to prospective undergraduates. Engineering students argued for a greater choice of modules as part of A-level Mathematics and, in particular, a greater choice of mechanics modules. In the study of physics undergraduates, it was noted that a high proportion of Further Mathematics students felt that studying it alongside A-level Mathematics, and by implication alongside Physics, was a sufficient preparation for the mathematics they studied as part of the physics degree. Moreover, Bowyer & Darlington (2016) observed that in their sample, students from more academically selective universities (such as those from the Russell Group) were significantly more likely to feel this way than from less-selective universities (e.g., non-Russell Group research universities and teaching-focussed universities). Among the motivations for studying Further Mathematics, prior enjoyment of, and attainment in school mathematics were noted as factors most often mentioned by the chemistry students. But the views of students on how well Further Mathematics fitted with their other subjects, and on how useful it would be in future, were also reported as motivating factors. The study of physics undergraduates noted that although Further Mathematics was not a formal entry requirement, the surveyed students perceived Further Mathematics as useful for their degrees and careers but the participants in academically selective universities felt more influenced by the perceived utility than those at less academically selective universities.

One may further hypothesize, if the support of parents, peers and teachers may be another factor influencing student choice as appeared to be the case among the mathematics undergraduates studied by the authors in Part 1 (Lyakhova & Neate, 2019). One other factor that should be considered is the format of the Further Mathematics course as the qualification is studied by small numbers of students at most schools may not be formally timetabled or may be taught through blended learning via the FMSP (Tanner *et al.*, 2016).

3. Methodology

A two-phase explanatory design was chosen for the study combining a student survey in Phase 1 with student interviews in Phase 2. This design was chosen so that statistical data for comparisons could be collected through the survey and quantitative data could be collected to inform the comparisons.

The survey was constructed to incorporate a range of factors which could influence student choice in studying mathematics. Commonly, emotions, cognition and motivation are considered as basic elements of the human mind which underpin behaviour in various situations including when making a choice to study mathematics (Schoenfeld, 1994). In the 1990s, McLeod (1992) proposed a highly influential framework of affect in mathematics with three main factors: emotions, attitudes and beliefs. This approach was later criticized for the lack of clarity in the definitions and interpretations and the aspects of effect which were not included (Hannula, 2011). While it is difficult to produce clearly defined notions of beliefs and attitudes, statements in relation to personal views and dispositions as well as to goals and desires (or motivations) are commonly used instruments for investigating mathematical behaviours (see, e.g., Tapia & Marsh, 2004; Diego-Mantecon *et al.*, 2007; Roesken *et al.*, 2011). In our survey, we used a 5-point Likert scale to record responses to statements in relation to mathematics self-concept (Marsh, 1986), previous experience of doing mathematics, teacher/parental influence (Ma, 2001; Mensah *et al.*, 2013) and views of the utility of mathematics. We further distinguished between student views about mathematics as a whole and their views on A-level Mathematics and Further Mathematics as qualifications. In our approach, we follow the views of Hannula *et al.* (2005) that in relation to mathematics, one can distinguish between the views of mathematics education, self and social context. The questionnaire contained 36 questions and can be found in Part 1 of this study (Lyakhova & Neate, 2019).

In Phase 1, the survey was conducted using 377 undergraduate students from 7 universities in the UK who were studying for the first year of a STEM degree. From these students, 367 who had studied at least AS-level Mathematics in England or Wales were selected for analysis. We primarily consider the results of 131 students in the survey who were studying for a non-mathematics degree. These students were studying for a degree in an area of either engineering or physics. The students were drawn from three universities selected to cover the diverse nature of universities within the UK (one from the Russell Group of traditional research universities, one from a non-Russell Group research university and one from a modern teaching-focussed university). Of these students, 126 had studied for the full A-level in Mathematics (the remaining 5 having completed only AS-level Mathematics) with 35 having studied for Further Mathematics in some form (17 studying AS-level Further Mathematics, 18 studying A-level Further Mathematics).

The students in this sample were predominantly male (85% male, 15% female). For comparison, across all universities in the UK in 2017–2018, 18% of the students studying in Engineering and Technology subjects were female (HESA, 2019). Of those in our sample who took Further Mathematics in some form, 29% were female.

As in Part 1 of this study, a named contact at each university who had agreed to assist with the survey was sent paper surveys with instructions to distribute them in a lecture of first year undergraduate students within physics and engineering. It was requested if possible that students were given time in the lecture to complete the survey to avoid a self-selecting sample.

Statistical analysis of the data was conducted using non-parametric tests as the Likert scale data from the survey is ordinal. The Mann–Whitney test was used for comparisons between two groups with significance level $\alpha = 0.05$. Students who had studied for at least AS-level Further Mathematics were classified as having participated in Further Mathematics.

There has been much controversy over the use of p values (Wasserstein & Lazar, 2016), and it should be noted that p values alone are not a demonstration of the existence of a difference or of the importance of any differences highlighted between groups. Rather, a small p value is evidence that the data collected suggest that there may be a difference in our comparisons. To illustrate the differences, graphs are included for comparisons between groups and an estimate of effect size $r = Z/\sqrt{N}$ (where N denotes the sample size) was calculated with effect sizes taken as 'small' ($r = 0.1$), 'medium' ($r = 0.3$) and 'large' ($r = 0.5$) (Cohen, 1988).

In Phase 2, semi-structured interviews (Bradford & Cullen, 2012) were conducted with undergraduate students studying for non-mathematics STEM degrees. As with Part 1, students from the survey sample (who had indicated they would be happy to be contacted) were initially approached for interviews. Due to a poor response rate to this approach, students who had previously taken part in the FMSP Wales engagement programme and who now studied at university were approached by email about taking part in the interviews. From this approach, 11 students responded and 8 were interviewed including 4 students who studied Further Mathematics. Two of the Further Mathematics students had studied the qualification through the FMSP. The students came from six universities including two Russell Group Universities. Degree courses included Physics, Engineering, Science with Economics, Computer Science and Earth Science. The universities involved in Phase 2 were not all involved in Phase 1. Participation in the interviews was voluntary and the participants were free to withdraw at any time during the interview.

The interviews were conducted on phone or in person at the end of participants' first year at university. The interviewer had a set of open-ended questions prepared in advance (Appendix 2). Students were asked about their degree course, about doing mathematics as part of their degree and about their A-level choice. This provided opportunities for students to make general comments about mathematics, about themselves doing mathematics at school and at university and about A-level choice of Mathematics and Further Mathematics. Thematic analysis (Braun & Clarke, 2006) was used for analysing the interview data. Identified recurring themes related to the A-level choice included students' recollections of choosing A-level Mathematics and/or Further Mathematics while at school, their views on the impact of studying or not studying Further Mathematics on their undergraduate study and their evaluation of their A-level choices from the position of an undergraduate student.

Ethical approval for the use of both the questionnaire and the interviews was sought and given prior to the commencement of the study. Free and informed written consent was sought from all participants. The research was conducted in accordance with BERA guidelines (BERA, 2011).

4. Results

We present the results over three sections beginning with a statistical analysis of the responses to the survey (Section 4.1) followed by a thematic analysis of the interviews (Section 4.2). We then reflect on the findings of both Parts 1 and 2 of the study in Section 4.3.

4.1. Findings from the survey

4.1.1. Mathematics and the choice of Further Mathematics. Almost all students in the survey gave positive responses about mathematics with some significant differences noted between those who chose to study Further Mathematics and those who did not (Fig. 1; Table A1). The students who studied

TABLE 1. *Non-Mathematics STEM undergraduate student sample by school exam results with national results for comparison (JCQ, 2017)*

Qualification		A*	A	B	C	D	E	U
A-Level Mathematics	Sample	11%	26%	42%	17%	2%	1%	0%
	UK	18%	24%	22%	16%	11%	6%	3%
GCSE Mathematics	Sample	44%	48%	7%	1%	0%	0%	0%
	UK	2%	3%	6%	22%	29%	19%	20%

NB: GCSE Mathematics is studied by all students in England and Wales typically before the age of 16 years.

A-Level passes are graded A*-E.

GCSE passes are graded A*-G. In this table grade U includes grades F, G and U.

Further Mathematics were significantly more likely to say that they enjoyed doing mathematics and were intending to study mathematics to the highest level they could. Further Mathematics students were also significantly more likely to view mathematics as their best subject and to describe themselves as 'naturally gifted at mathematics'. These were medium-size effects.

A large majority of these students had experienced early success with Mathematics with 92% gaining an A or A∗ in GCSE mathematics (Table 1). This was reflected in their perceptions of GCSE where 82% agreed they had found GCSE Mathematics easy. However, there were more diverse experiences in the study of mathematics post-16. Only 33% agreed that they had found AS-level Mathematics easy and 63% perceived that there had been a bigger jump in difficulty between GCSE and AS-level in Mathematics than in their other subjects. However, those who had studied Further Mathematics were significantly more likely to say that they had found AS-level Mathematics easy.

Most of the respondents felt that hardwork was more important than ability in mathematics with 67% agreeing to this statement and only 3% disagreeing. Similarly, only 9% of the respondents agreed that only gifted mathematicians should study A-level Mathematics and the majority (55%) of all the students disagreed with the statement.

The participants generally felt supported by their teachers in their choice of studying mathematics post-16 with 76% of all students agreeing that their teachers had encouraged them to study AS/A-level Mathematics. However, Further Mathematics students were significantly more likely to feel supported by teachers in their choice AS/A-level Mathematics than those students who did not choose Further Mathematics.

In comparison, when asked about whether they had been discouraged by their teachers from studying Further Mathematics, 51% disagreed but 32% were neutral suggesting they were unsure or the matter had never been discussed with them. While a greater proportion of those who took Further Mathematics strongly disagreed with this statement (37% versus 13%) the difference in the distribution of responses was not statistically significant ($Z = 1.067$, $p = 0.286$).

Similarly, the majority of students felt supported by their parents in their choice to study Mathematics with 64% agreeing that their parents thought that it was important to study at least AS-level Mathematics. But there was a considerably smaller perception of parental support for studying the Further Mathematics qualification, with 50% unsure on how their parents felt about Further Mathematics. Although the Further Mathematics students were significantly more likely to feel encouraged by their parents, only 20% of these students felt that their choice of Further Mathematics was important to their parents.

While a lack of encouragement from teachers and parents could have been the factors that lead some students to not study Further Mathematics, the survey data showed that several other reasons might also be important (Fig. 2; Table A2). Among the students who did not study Further Mathematics,

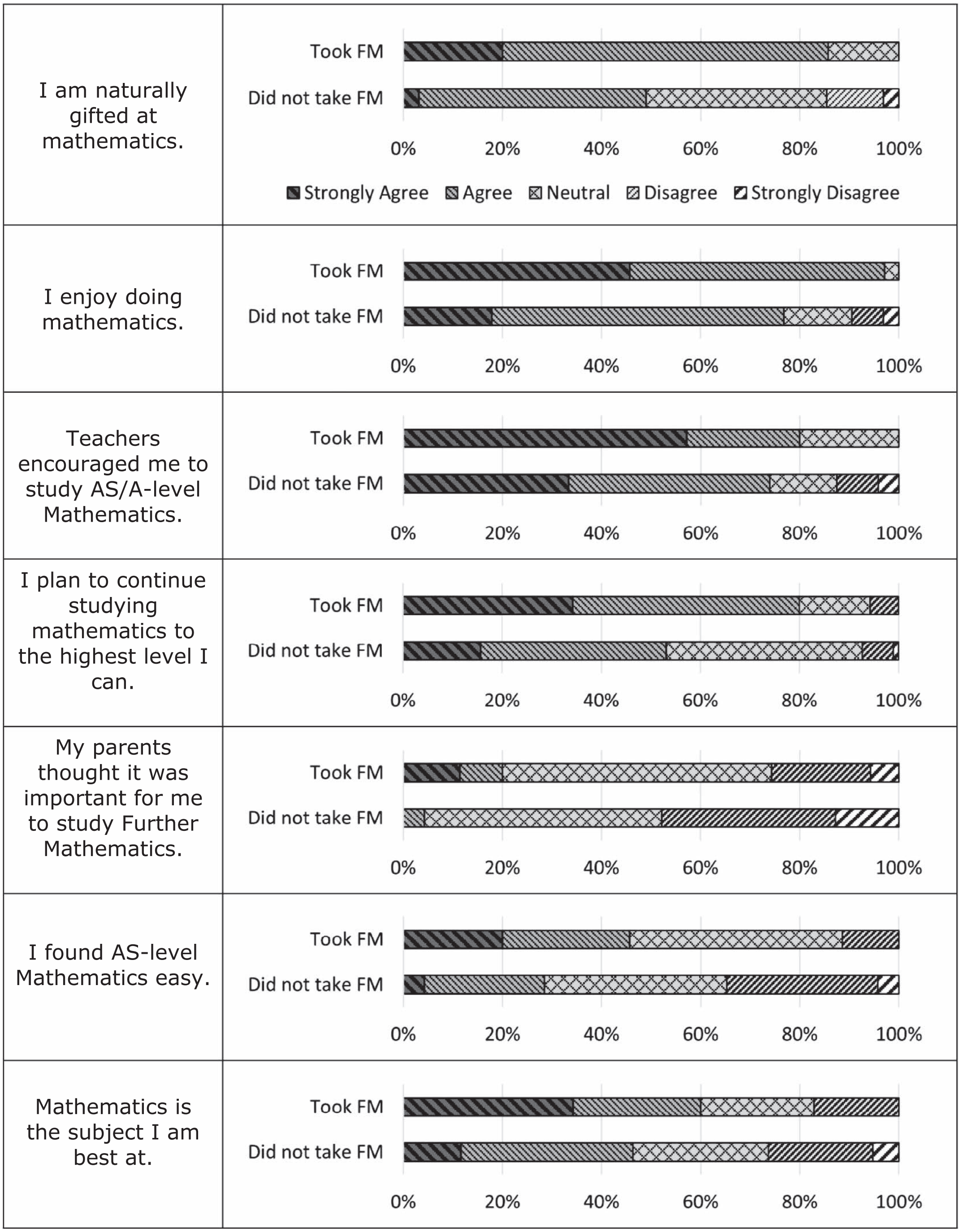

FIG. 1 Selected significant differences (Mann–Whitney) between those who had and had not studied Further Mathematics among non-mathematics STEM undergraduate students.

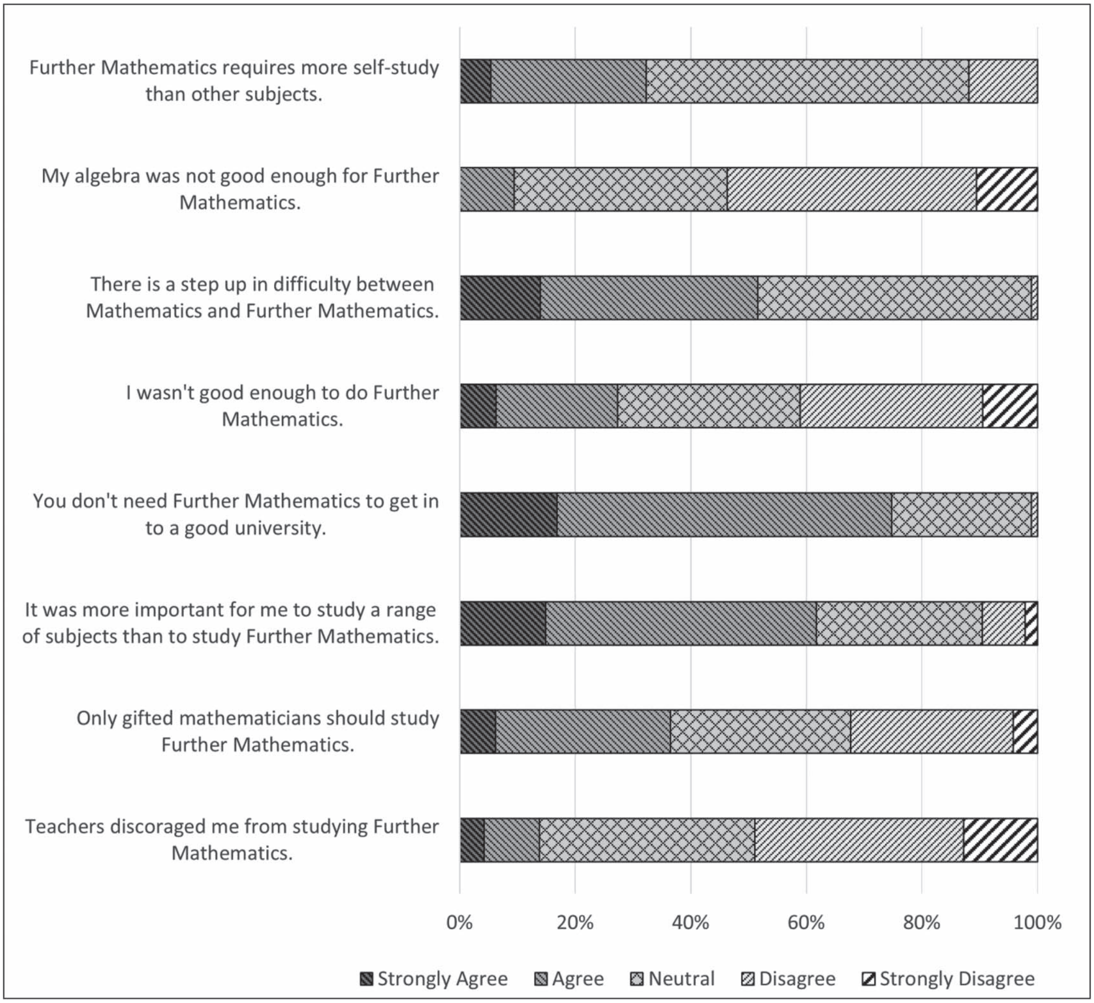

FIG. 2 Non-Mathematics STEM undergraduate students who did not take Further Mathematics, reflecting on their motivations for not choosing it.

a high proportion (75%) did not associate studying Further Mathematics with helping to ‘get into a good university’. A substantial proportion (27%) felt they did not have the ability to cope with Further Mathematics. This could have been underpinned by a range of views including that studying Further Mathematics only suited gifted students (37% agree), that the subject was more demanding in comparison with A-level Mathematics (52% agree) and that it required more self-study than other subjects (32% agree). A large proportion (62%) of the students who did not study Further Mathematics agreed that they felt studying a range of subjects was more important than studying Further Mathematics.

There was also evidence of a perceived lack of utility in Further Mathematics. Among the students who had not studied Further Mathematics, 76% agreed that having studied at least AS-level Mathematics helps if you apply for university but only 39% agreed that studying Further Mathematics would help.

Similarly, 58% agreed that having at least AS-level Mathematics helps to get a job but only 33% felt similarly about Further Mathematics. In both of these questions, the greatest response about Further Mathematics was 'Neutral' (46% and 47%, respectively).

Some of the perceptions of those who did not study Further Mathematics were confirmed by the experiences of those who studied for the qualification. Further Mathematics students were in agreement about Further Mathematics being a demanding qualification, with 91% of these students agreeing there was a step up in difficulty between A-level Mathematics and Further Mathematics and 53% agreeing that Further Mathematics required more self-study than other subjects. In contrast to how students felt about studying A-level Mathematics, 51% of those who studied Further Mathematics felt that only gifted students should study the subject. While there were no significant differences between Further Mathematics and non-Further Mathematics students about the importance of Further Mathematics qualifications for helping to 'get into a good university' ($Z = 1.60$, $p = 0.11$) or getting a job ($Z = -1.41$, $p = 0.16$), more than half of the students who had studied Further Mathematics (54%) agreed that having a Further Mathematics qualification helps to 'get into a better university'.

4.1.2. Further Mathematics and the transition to university. In the survey, 85% of the students who studied Further Mathematics agreed that they perceived that it had made their transition from school to university mathematics easier and 44% agreed that they felt they would have struggled to cope with the mathematics in their course if they had not studied Further Mathematics. Moreover, 91% of these agreed that studying Further Mathematics broadened their understanding of mathematics in general with no Further Mathematics students disagreeing with the statement. In comparison to those who did not study Further Mathematics, there is evidence of a perceived advantage when transitioning to studying mathematics at university with 34% of Further Mathematics students agreeing they have to work hard just to keep up, compared with 52% of those who did not. However, the difference between these two groups was not significant ($Z = 1.67$, $p = 0.095$).

The survey data also suggest that studying Further Mathematics is associated with differences in intentions towards the future study of mathematics. Further Mathematics students were significantly more likely to agree that they were planning to continue studying mathematics to the highest level they could (Fig. 1; Table A1).

A perception of the helpfulness of Further Mathematics was also seen among the students who did not take Further Mathematics. In the survey, 41% of those who did not take Further Mathematics now wished they had.

While Further Mathematics was perceived as beneficial, we do not have any evidence of a change in approach to mathematics. Among all students, 48% agreed they found it easier to memorize mathematics techniques than those who preferred to try to understand them with no significant difference between those who did and did not study Further Mathematics ($Z = 0.98$, $p = 0.33$).

4.1.3. Gender differences. Only a few gender differences were noticed in the sample (Fig. 3). Girls studying a non-mathematics STEM degree were more likely to consider themselves gifted in mathematics and to enjoy mathematics more. Perhaps, girls also enjoyed it in a different way as they were significantly more likely than boys to say that they enjoyed the certainty of mathematics. All of these were small to medium effects.

4.2. *Findings from the interviews*

We present the findings of the interviews based on our thematic analysis.

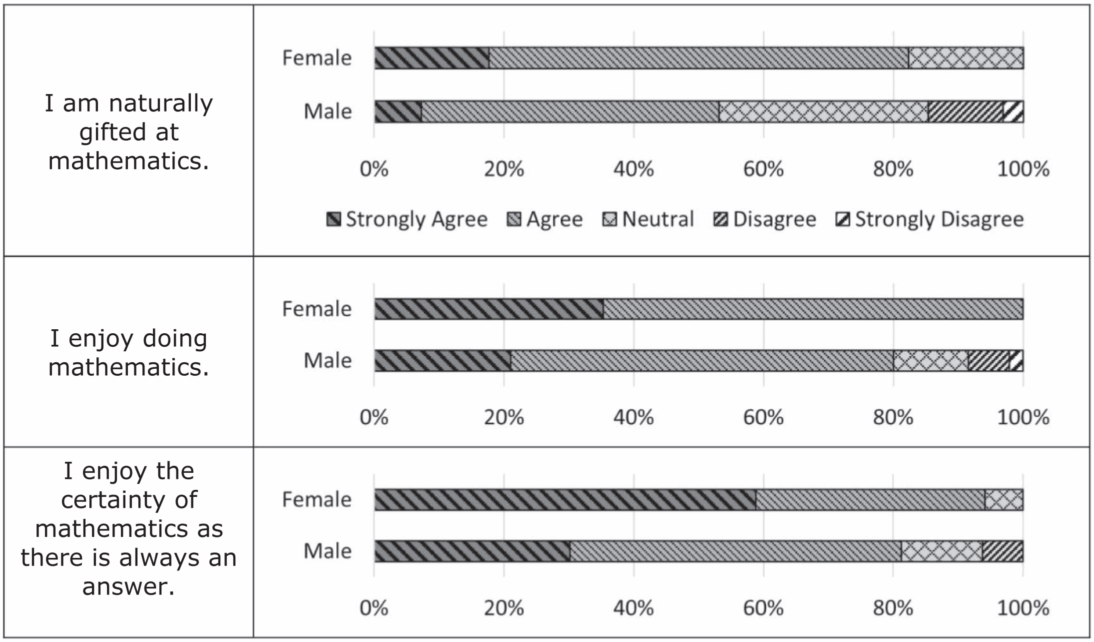

FIG. 3 Significant differences (Mann–Whitney) between genders among non-Mathematics STEM undergraduate students.

4.2.1. Mathematics at pre-16. Consistent with the survey sample, all the interviewed respondents displayed an interest in, and enjoyment of, mathematics. They generally felt that their interest and enjoyment started very early, perhaps in primary school, where they enjoyed working with numbers and solving puzzles. One student explained how his perception of mathematics had formed: ‘I think when you are younger, anything you are good at you like doing more really. Then you get praised by the teachers and so I enjoyed doing [mathematics] even more and proving I can do more [mathematics] than [the teachers] gave me’. We noted that students associated their pre-16 experiences of enjoying mathematics with finding mathematics easy.

4.2.2. Choosing A-level Mathematics and Further Mathematics. The choice of A-level Mathematics was straightforward for all respondents in our sample but for a variety of different reasons. One student had considered applying to study for a mathematics degree and so saw A-level Mathematics as essential. But taking A-level Mathematics was also perceived as helpful for keeping options open when choosing between degree courses other than mathematics. One boy explained, ‘I might have gone into law, so history would back that up. Then if I went to computer science then maths would back that up’. For those who already targeted ‘a degree with a maths related element’, studying mathematics also meant looking at ‘more prestigious’ universities. In relation to this, some students took pride in choosing A-level Mathematics as part of a ‘hard’ A-level subject combination, alongside, for example, Physics and Chemistry. However, the students only mentioned A-level Mathematics (and not Further Mathematics) in these contexts. The rationale for choosing Further Mathematics seemed to be different.

Among the students who studied Further Mathematics the decision to take this subject was made because of teacher encouragement. Notably, the decision was made later than the decision to study A-level Mathematics. For example, one girl explained, ‘My maths teacher told me about [Further Mathematics]. I thought – extra maths? I’ll do that!’ and she swapped a different subject for Further Mathematics within the first month. Studying AS-level Mathematics was referred to as a decisive point regarding choosing Further Mathematics. Indeed, either students felt prompted by their teachers to look at Further Mathematics when they realized that they coped well with AS-level Mathematics or good AS-level Mathematics results gave them reassurance that they would be able to continue with Further Mathematics despite it being a fourth or fifth subject.

The two students who studied Further Mathematics through FMSP emphasized that their commitment was not only to do more mathematics but also to exert an extra effort on their side even though they felt they were ‘working to [their] strengths’. The student who studied a full A-level through FMSP stressed that one has ‘to be quite committed to do Further Maths. The lessons are quite sporadic and you have to do a lot of independent work’. Another student who did AS-level Further Mathematics in year 13 explained that the self-study element required good organizational skills as ‘you are more prone to distraction at home’. When making the Further Mathematics choice, the students felt reassured that they would be supported by their school even though they were to receive external tuition. One student, for example, said that his ‘school had a good reputation for maths’ and that was why he expected to get helped.

Among the students who did not have a Further Mathematics qualification, there was a perception that Further Mathematics was ‘something that people do outside the curriculum’ and that studying it involved a lot of extra work. At the same time, none of the universities these students considered mentioned Further Mathematics. The subject was not offered at the schools these students attended and evidently their teachers did not mention it as an option. As a result, the students felt they ‘never gave it a thought’.

The role of advice from teachers was also noted for subjects other than Further Mathematics, especially when directing students to investigate the subjects they were interested in beyond the school programme. The students were also consistent in their remarks about AS-level being a convenient point to ‘swap’ other subjects and not just Further Mathematics.

4.2.3. Studying mathematics at university. All the interviewed students found the mathematical components of their degrees demanding. If a student studying Physics expected his/her course to contain a lot of mathematics, students studying university degrees as varied as computer science, economics, engineering and earth science reflected how they felt genuinely surprised at the amount of mathematics in the first year of their degree. There was a feeling that university mathematics was different from A-level ‘as it is SO applied’. The students felt that their degree courses required them to pick up techniques and methods quickly, without dwelling on understanding. As one student explained it, an undergraduate mathematics module would ‘not teach you the maths that underpins it. You know what to do but do not know why’. Deciding about which methods to use when solving problems was seen as another challenging element both by those students who studied Further Mathematics and those who did not. Studying more pure mathematics modules and at slower pace, that is, ‘like in school’, was proposed by some students as a way to improve their skills.

A few remarks were made about other students on their courses who did not study A-level Mathematics and were required to do an extra mathematics module as part of their degree. One of the students explained what he thought was the reason for some students in his course not studying an A-level Mathematics: ‘I suppose people at the start of A-level don’t know the degree course they are going to want to do’’. However, he also remarked that A-level Mathematics was not a requirement for his course either.

It was evident from the interviews that the students generally felt an effort on their part was needed to study mathematics at university and that their experience proved that the effort would bring the desired outcomes. ‘The work will get difficult, so you have to motivate yourself to get through it’ emphasized one student as part of her advice to students considering a mathematics-based degree at university. Yet, ‘actually most of the time if you apply yourself, you will be fine’ stressed another student.

4.2.4. The impact of studying or not studying Further Mathematics. The content of Further Mathematics modules was seen as directly relevant to the mathematics content of the degree courses. One of the students who studied Further Mathematics remarked: ‘I do not think we used much standard A-level maths in the first year, but the Further Pure stuff on matrices we did. We did loads of proof by induction. I can’t remember doing that in regular maths, I did it in Further Maths’. We observed a similar attitude in students who had not studied Further Mathematics. ‘The first year [undergraduate] maths was all Further Maths stuff. I had not realised the maths would be at such a high level so quickly,’ said one student about her geophysics course. Students felt that not only the university teaching was related to the topics studied in Further Mathematics but also that studying similar topics as ‘pure’ mathematics in school gave them a better understanding of how to apply this mathematics. ‘You just know how it works. That’s it!’

Being familiar with such Further Pure Mathematics topics as matrices, vectors, proof by induction and complex numbers allowed students to concentrate on the problem-solving aspects of their courses. But the same topics were mentioned by the students who did not study Further Mathematics. For example, the Physics undergraduate remarked: ‘If you don’t do Further Maths, you don’t get to see matrices at all. The first year was all about matrices, how to transpose them . . . [Further Mathematics] would have helped a lot.’ He further generalized how he saw the impact of not studying Further Mathematics on his first year at university: ‘Most of [my time] was spent getting to grips with the maths rather than the actual physics concepts. If I’d had a better understanding of the maths to start with it would have been a lot easier.’

In the survey, we observed that studying Further Mathematics was associated with students’ intentions to study more mathematics. This was also observed in the interviews, where two Further Mathematics students (both girls), reflected with great enthusiasm how they picked more mathematics modules as they went along with their studies. As we noted earlier, one student when prompted by her teacher about studying Further Mathematics said, ‘extra maths? I’ll do that!’, and this student carried over this attitude into her university studies. ‘In my first year I could pick extra modules and I chose more maths—why not?’ she said laughing happily. Another student studying engineering explained, ‘There is loads of maths in my [assignment], but this is my choice.’ Both students felt that they always (‘even at primary school’) enjoyed mathematics and wanted to do extra mathematics (‘puzzles and anything to do with numbers’) as well as both having a high perception of their mathematical ability (‘I was always ahead’). However, these students evidently were also prepared to work hard to succeed with their study. ‘Someone once told me “a good degree is a hard degree”, so I’ve kept this in mind’, noted one of the girls.

4.2.5. Reassessing A-level choices once at university. Reflecting on their A-level choice, all the students universally agreed that studying mathematics generally was the most useful preparation for their courses. One student went as far as to say that A-level Mathematics was the only subject directly applicable to his degree course in engineering. He felt that mathematical content of his degree was building up directly from the content of A-level Mathematics, whereas the content of other A-level subjects was covered as part of his university course.

Several students in our sample chose their subjects very specifically but those who felt that their choice provided them with a balanced set of skills felt the happiest. For example, a girl who was studying

Computer Science considered her choice of A-level Mathematics, A-level Chemistry, AS-level History and A-level Further Mathematics was ‘the best preparation’ for her degree. She felt that a combination of studying more mathematics (Further Mathematics) as well as History gave her a lot of qualitative skills as well as prepared her for the writing element of her degree. ‘I would not recommend three sciences. I’d say drop the science you like least and do one AS in humanities,’ commented the student. However, she, like all other interviewed students with Further Mathematics qualifications, admitted that she would not drop Further Mathematics because ‘mathematics is underlying all the work in Computer Science’.

This view of Further Mathematics was confirmed by the comments from the students who did not choose to study it. One of these students explained that although she enjoyed her choice of A-level subjects at the time, she would now swap one subject for Further Mathematics as the other subjects she had studied were ‘a bit like using everyday knowledge you have [anyway]’. In comparison, she felt that mathematical knowledge could not be easily acquired without actually studying it. She now wished she had studied Further Mathematics even if not sitting the exam; she had previously dropped the Further Mathematics course partially because of the workload but also because she did not think the course was applicable to her degree choice.

Additionally, we registered a feeling of surprise among the Further Mathematics students about how useful Further Mathematics proved to be. This is perhaps understandable given that Further Mathematics was not a requirement for their degree course and the students did not choose it specifically for the course. One Further Mathematics student said, ‘You would think that you do Further Maths for a maths degree’ stressing that ‘people who hadn’t done Further Maths in the first year really struggled’.

4.3. A comparison of the surveys between mathematics (Part 1) and non-mathematics undergraduates (Part 2)

Here, we briefly compare the survey data presented in Part 1 of this study, with that presented in Part 2. This involved data from the full sample of 367 students. We consider students who studied Further Mathematics and those who did not separately. In each of these groups, there are several significant differences between the mathematics and non-mathematics undergraduate students which we reproduce in Figs 4 and 5 (see also Tables A3 and A4).

Among the Further Mathematics students, the differences indicate that mathematics and non-mathematics undergraduates had different perceptions about mathematics, possibly, underpinned by experiencing AS-level Mathematics as easier or more difficult. But they also had different views on whether Further Mathematics helps to ‘get into a better university’. Interestingly, more than 40% of non-mathematics undergraduates neither agreed nor disagreed that they found AS-level Mathematics easy.

Among the non-Further Mathematics students, those who studied for a non-mathematics degree were significantly less likely to have found studying for AS-level Mathematics easy, which was a large effect. They were also less likely to enjoy mathematics or consider it to be their best subject (both medium–large effects). They were less likely to view themselves as naturally gifted in mathematics (small effect). These students were also less likely than mathematics undergraduates to agree that only gifted mathematicians should study A-level Mathematics. In either group, only a small minority of students agreed that studying Further Mathematics was considered as important by their parents but non-mathematics undergraduates were significantly more likely to disagree with the statement. In fact, none of the non-mathematics students strongly agreed with that it was important for their parents while there were some among the mathematics undergraduates.

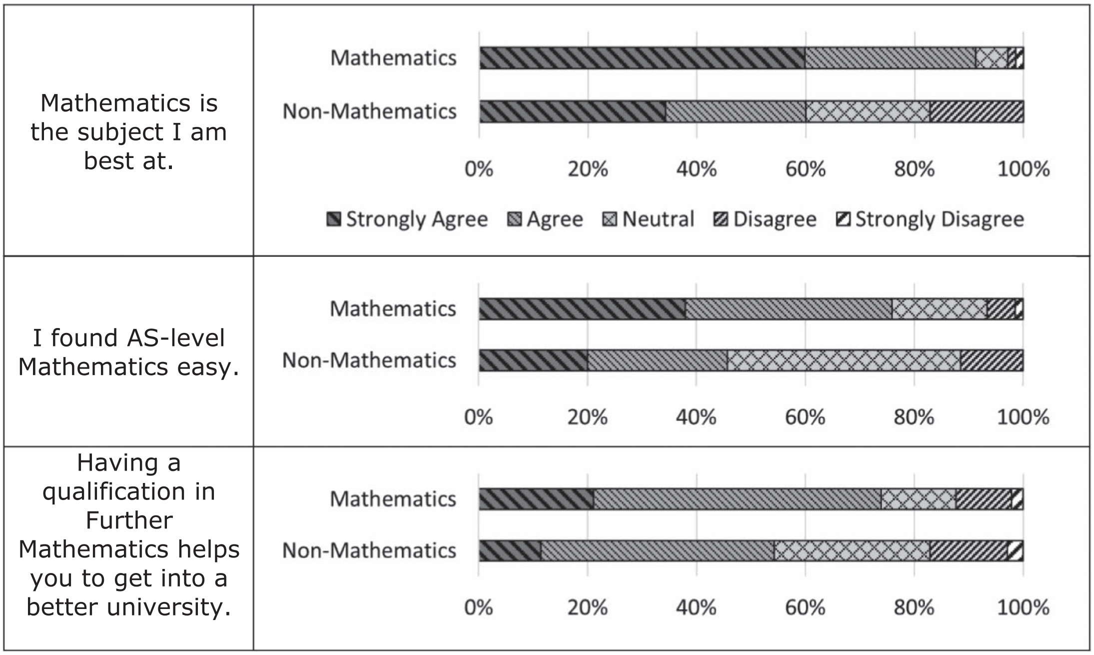

FIG. 4 Selected significant differences (Mann–Whitney) between mathematics and non-mathematics STEM undergraduate students who had studied Further Mathematics.

5. Discussion

Further Mathematics is perceived by students as useful preparation for studying non-mathematics STEM degrees at university. Among those students that studied Further Mathematics our findings identified a perception that studying Further Mathematics made the transition from school to university mathematics easier. Moreover, many Further Mathematics students perceived they would have struggled to cope with the mathematics in their course if they had not studied the subject. There were several different aspects to the perceived impact of studying Further Mathematics. Further Mathematics was seen as helping with the mathematical content as well as enabling the students to focus on the non-mathematical aspects of their course while also giving them the confidence to pick more optional mathematics modules at university. One may agree with Schoenfeld (1994) that the latter (that is, when learners are ready and enthusiastic to do more mathematics) is the very aim of mathematics education. In this respect, these learners could be seen as succeeding in learning mathematics.

But, among the students who did not study Further Mathematics, many in retrospect wished they had. While we cannot assume that the students who did not study Further Mathematics were aware of what a Further Mathematics course involved, we can speculate that some of these students reached this conclusion based upon how useful they found Mathematics in comparison with some other A-levels they chose. This in turn could be related to the fact that the mathematics components were perceived as challenging due to the fast pace and advanced nature of the mathematics the students were expected to learn and apply rapidly in their degree course.

Although it is difficult to distinguish between the impact of studying more mathematics and studying Further Mathematics *per se*, it seems that the content of the Further Mathematics course

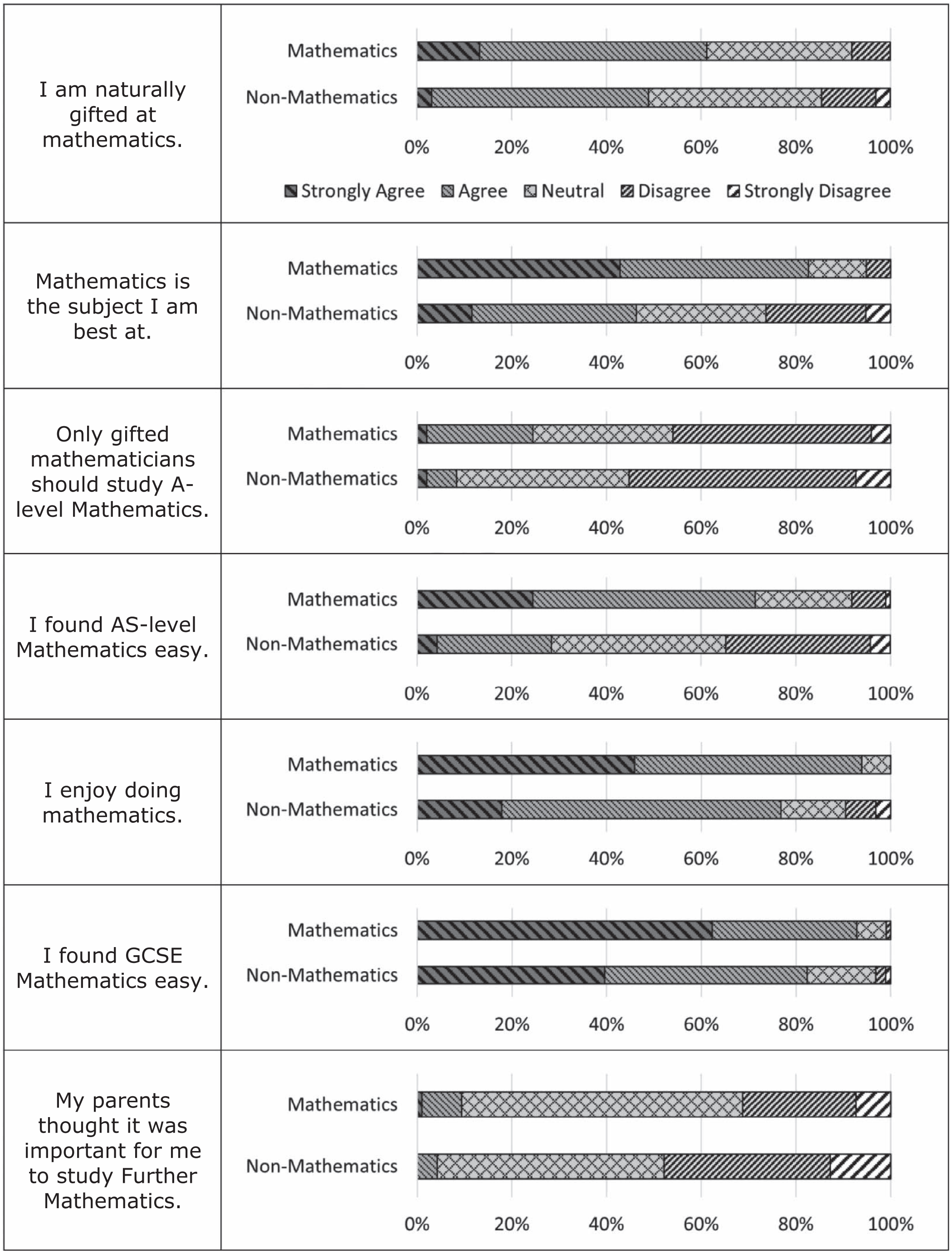

FIG. 5 Selected significant differences (Mann–Whitney) between mathematics and non-mathematics STEM undergraduate students who had not studied Further Mathematics.

plays an important role for the transition to university. The interviewed students accentuated that pure mathematics modules were most useful despite the degree courses using applied mathematics. It was suggested that studying more pure mathematics helped to improve their understanding of how to apply mathematics. This may explain why the vast majority of Further Mathematics students agreed that studying Further Mathematics broadened their understanding of mathematics in general. This confirms and extends the previous findings of Darlington and Bowyer (2016a, 2016b, 2017) on the perceived usefulness of pure mathematics topics for studying non-mathematics STEM degrees. This is of importance as it questions a popular view that was noted to influence reforms of post-16 mathematics qualifications (Hoyles *et al.*, 2001), which suggests that making school mathematics too relevant to mathematics that is studied at the level of a mathematics degree (in other words, too 'pure') may reduce its perceived utility for students who are considering different pathways.

An intention to study more mathematics in the future could be interpreted as increased confidence in their ability to tackle more, and more difficult, mathematics building from Further Mathematics and, indeed, we have some evidence from the interviews to support this. However, we also have evidence from both survey data and the interviews that students may already have had a high perception of their mathematical ability and a positive approach to mathematics when they chose Further Mathematics. Yet, these seem to not be enough for students to choose Further Mathematics as our study shows. The findings imply that the choice of A-level Mathematics was straightforward for many with this decision underpinned by a previous positive experience of studying mathematics, the support of teachers or parents and the perceived high utility of the qualification. On the contrary, the evidence we considered makes it difficult to hypothesize that the latter two factors were equally influential for Further Mathematics choices. For example, the usefulness of Further Mathematics for STEM degrees surprised even those students who studied Further Mathematics. But, more generally, a view noted among the non-mathematics undergraduates that Further Mathematics is only for those studying for a mathematics degree may be seen as confirmed by our findings on the comparison between mathematics and non-mathematics undergraduates but this view may also have been shared by students' parents. This is perhaps unsurprising given that universities generally do not promote the study of Further Mathematics for non-mathematics degrees, and has been previously noted, there is a lack of awareness of the qualification among parents (Tanner *et al.*, 2016).

Among the positive views about mathematics among the respondents in this study included a positive perception of the role of effort in learning mathematics. Indeed, most of the survey respondents felt that hard work was more important than ability in mathematics and in the interviews, all students expressed a view that effort was essential to cope with the mathematical content of their degree course. Since GCSE, many students seem to have met mathematics that they did not find easy. However, our findings also point out that such an attitude did not transfer into the attitudes towards studying a Further Mathematics qualification. As we found, a perception prevails that studying Further Mathematics depends on one's mathematical ability. This has been noted among both the groups who studied Further Mathematics and those who did not. It is nevertheless unclear what students perceive as natural ability in mathematics and how they view the relationship between their ability and effort.

The arguments above encourage one to hypothesize that the advice of teachers (or a lack of it) is, perhaps, the most influential factor in the context of Further Mathematics for students who are considering non-mathematics degrees. The interview data suggest strongly that teachers played an important role in suggesting Further Mathematics or making it available (such as arranging students' access to studying it through the FMSP). Yet, the survey data did not indicate a statistically significant difference in students' perceptions of the support teachers gave with regard to Further Mathematics, only with regard to Mathematics. The data do show a difference, however, and the lack of statistical significance may be the result of the small sample size of students who took Further Mathematics. This still implies that students who choose Further Mathematics are more likely than students who did not, to feel supported by their teachers in relation to post-16 studies of mathematics (if not Further Mathematics itself). But we would like to speculate that the relationship between AS-level Mathematics and the choice of Further Mathematics may not be random, after all; the experience and results of AS-level Mathematics (and possibly other subjects) was evidently used by students to evaluate their progress and make decisions about Further Mathematics. Importantly, we have evidence that such decisions were made in discussion with teachers. More investigation into what the advice teachers give is based upon would be beneficial.

The findings on the gender differences for non-mathematics STEM subjects could be interpreted as evidence of an equal level of self-confidence in mathematical skills among the girls and boys studying non-mathematics STEM degrees. Yet, boys may be more likely to choose to study a non-mathematics STEM degree than girls and STEM is still seen as a subject or career domain for boys (see, e.g., Shapiro & Williams, 2012). What our findings perhaps imply is that developing their mathematical skills at school could be a factor which gives girls some confidence to enter the STEM domain.

In our study, an association was noted between how well students cope with AS-level Mathematics and whether they take Further Mathematics and also between AS-level Mathematics and whether students choose mathematics or other STEM subject as a degree course. It was also registered that it is in AS-level Mathematics when many students first meet mathematics that they view as difficult. This implies that transition from pre-16 to post-16 study of mathematics may be of more importance for further transition to an undergraduate degree than has been considered in our study. But it also could be that performance at lower level of mathematics (e.g., at the age of 16 years) alone may be an important factor for succeeding with undergraduate study as, for instance, a study by Nicholas *et al.* (2015) found.

We remark that some studies of the transition to non-mathematics undergraduate degree courses question whether it is the actual mathematical knowledge and skill of the post-16 mathematics qualifications or, rather the higher order cognitive skills that studying mathematics may foster in students, that actually helps them to succeed at university level (see, e.g., Donovan & Wheland, 2009; Adkins & Noyes, 2018). Little is known about the specific cognitive skills that are developed through A-level Mathematics (although see Attridge & Inglis, 2013; Gilbey & Robson, 2017) or, for that matter, through A-level Further Mathematics. However, both parts of our study demonstrate that Further Mathematics is perceived as requiring more work, harder work or more commitment from the students (Lyakhova & Neate, 2019) which can be hypothesized to demand and/or foster cognition in students studying it.

Our findings also contain implications on both the structure of the A-level qualifications and the content. In the 2017–2018 reforms, Wales and England took different pathways in relation to how an AS-level qualification counts towards a full A-level qualification. While in Wales, a reformed AS-level counts as 40% towards a full A-level, in England a reformed AS-level is a stand-alone qualification. The first two years of results of the new reformed A-level qualifications showed a sharp decrease in the numbers of students taking AS-level Further Mathematics as well as AS-level Mathematics in England while in Wales the numbers remained fairly stable (JCQ, 2019). An AS-level being separate from a full A-level may mean that students no longer have the option to use their AS-level results to evaluate their progress, and they no longer have the opportunity, or incentive, to re-consider their A-level choices halfway through as they did before the 2017–2018 reforms. As our findings highlight, mathematics teachers need to find an opportunity to discuss intermediate progress no matter how the qualifications are constructed.

Our findings allow us to hypothesize that Further Mathematics (as studied before 2017) was serving those who went on to study for a non-mathematics STEM degree better than those who progressed to studying for a mathematics degree. The survey data in the studies of both mathematics (Lyakhova & Neate, 2019) and non-mathematics undergraduates did not register changes in approaches to mathematics learning related to Further Mathematics. However, we did not come across any arguments that a change in the approach of students was needed for adapting to the mathematical components of non-mathematics degree courses. In contrast, mathematics undergraduates studying pre-reformed Further Mathematics qualifications did not view Further Mathematics as enough preparation for proving and constructing mathematical arguments as well as problem solving which is seen as important within mathematics at university, which was argued to be linked to students adopting a surface approach to learning. This is perhaps unsurprising, given the fact that many former reforms of A-level qualifications were driven by demand to serve the needs of students who would be applying mathematics in the context of other disciplines (Hoyles *et al.*, 2001). Therefore, changes in the content to serve students applying to mathematics degrees may need to be prioritized as part of reforms. There are reasons to be optimistic about the changes that new reformed A-level qualifications could bring and the authors hope that future studies on the reformed qualifications will shed light on this.

In conclusion, Further Mathematics qualifications are perceived as useful for studying non-mathematics STEM degree courses. Moreover, non-mathematics undergraduates perceive less shortcomings of Further Mathematics in comparison with mathematics undergraduates. Ironically, however, they are less likely to study Further Mathematics as a result of perceiving that Further Mathematics is only for those studying a mathematics degree. This is perhaps underpinned by a general view that it is not necessary to study as much mathematics as possible to create a good foundation for STEM subjects. As our study demonstrated, positive perceptions of mathematics are different from their perceptions of the Further Mathematics qualifications which, when underpinned by the lack of advice from university admissions, teachers and parents, may prevent students from studying Further Mathematics.

Acknowledgement

As with Part 1, this study was initiated with Howard Tanner (University of Wales Trinity St David) who very sadly passed away before it could be completed. We would like to acknowledge his enormous contribution to the study.

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Appendix A. Tables of results

TABLE A1. *Selected significant differences (Mann–Whitney) between those who had and had not studied Further Mathematics among Mathematics undergraduate students*

Survey statements	Further Mathematics	Strongly agree	Agree	Neutral	Disagree	Strongly disagree	Sample size	p	Z	r
I am naturally gifted in mathematics.	Yes	20%	66%	14%	0%	0%	35	<0.000	−4.39	0.38
	No	3%	46%	37%	11%	3%	96			
I enjoy doing mathematics.	Yes	46%	51%	3%	0%	0%	35	<0.000	−3.74	0.33
	No	18%	59%	14%	6%	3%	95			
Teacher(s) encouraged me to study AS/A-level Mathematics.	Yes	57%	23%	20%	0%	0%	35	0.025	−2.24	0.20
	No	33%	41%	14%	8%	4%	96			
I plan to continue studying mathematics to the highest level I can.	Yes	34%	46%	14%	6%	0%	35	0.004	−2.91	0.25
	No	16%	38%	40%	6%	1%	96			
My parents thought it was important for me to study Further Mathematics.	Yes	11%	9%	54%	20%	6%	35	0.004	−2.88	0.25
	No	0%	4%	48%	35%	13%	94			
I found AS-level Mathematics easy.	Yes	9%	26%	43%	11%	0%	35	0.003	−2.94	0.26
	No	4%	24%	37%	31%	4%	95			
Mathematics is the subject I am best at.	Yes	34%	26%	23%	17%	0%	35	0.025	−2.24	0.2
	No	12%	35%	27%	21%	5%	95			

TABLE A2. *Significant differences (Mann–Whitney) between genders among non-Mathematics STEM undergraduate students*

Survey statements	Gender	Strongly agree	Agree	Neutral	Disagree	Strongly disagree	Sample size	p	Z	r
I am naturally gifted at mathematics.	M	7%	46%	32%	12%	3%	96	0.013	−2.49	0.23
	F	18%	65%	18%	0%	0%	17			
I enjoy doing mathematics.	M	21%	59%	12%	6%	2%	95	0.043	−2.03	0.18
	F	35%	65%	0%	0%	0%	17			
I enjoy the certainty of mathematics as there is always an answer.	M	30%	51%	13%	6%	0%	96	0.020	−2.33	0.22
	F	59%	35%	6%	0%	0%	17			

TABLE A3. *Significant differences (Mann–Whitney) between Mathematics and non-Mathematics undergraduates among those who had studied Further Mathematics*

Survey statements	Mathematics UG	Strongly agree	Agree	Neutral	Disagree	Strongly disagree	Sample size	p	Z	r
Mathematics is the subject I am best at.	Yes	60%	31%	6%	2%	2%	137	<0.000	−3.65	0.28
	No	34%	26%	23%	17%	0%	35			
I found AS-level Mathematics easy.	Yes	38%	38%	18%	5%	2%	137	0.003	−3.01	0.23
	No	20%	26%	43%	11%	0%	35			
Having a qualification in Further Mathematics helps you to get into a better university.	Yes	21%	53%	14%	10%	2%	138	0.035	−2.11	0.16
	No	11%	43%	29%	14%	3%	35			

TABLE A4. *Significant differences (Mann–Whitney) between Mathematics and non-Mathematics undergraduates among those who had not studied Further Mathematics*

Survey statements	Mathematics UG	Strongly agree	Agree	Neutral	Disagree	Strongly disagree	Sample size	p	Z	r
Mathematics is the subject I am best at.	Yes	43%	40%	12%	5%	0%	98	<0.000	−6.10	0.44
	No	12%	35%	27%	21%	5%	95			
Only gifted mathematicians should study A-level Mathematics.	Yes	2%	22%	30%	42%	4%	98	0.033	−2.13	0.15
	No	2%	6%	37%	48%	7%	96			
I found AS-level Mathematics easy.	Yes	25%	47%	20%	7%	1%	98	<0.000	−6.38	0.46
	No	4%	24%	37%	31%	4%	95			
I am naturally gifted at mathematics.	Yes	13%	48%	31%	8%	0%	98	0.016	−2.42	0.17
	No	3%	46%	37%	12%	3%	96			
I enjoy doing mathematics.	Yes	46%	48%	6%	0%	0%	98	<0.000	−4.80	0.35
	No	18%	59%	14%	6%	3%	95			
I found GCSE Mathematics easy.	Yes	62%	31%	6%	1%	0%	98	0.001	−3.34	0.24
	No	40%	43%	15%	2%	1%	96			
My parents thought it was important for me to study Further Mathematics.	Yes	1%	8%	59%	24%	7%	96	0.012	−2.52	0.18
	No	0%	4%	48%	35%	13%	94			

Appendix B. Interview questions

1. You have done a lot of mathematics since you finished GCSE. How and why did you make this decision? What influenced your decision?
2. Tell me about the degree you are studying. Why did you decide to do it?
3. How much mathematics does your degree course involve? Which aspects are the most challenging for you and why?
4. What influenced you to study a mathematics-based subject at a university? When did you make this decision?
5. What did you study at A-level which has helped you in this course? What do you think about Further Mathematics A-level?
6. How is studying mathematics at university different from A-level Mathematics? What about Further Mathematics?
7. What advice would you give to someone who is thinking of doing a mathematics-based degree at a university?